EASY
1. Determine the time required to charge a capacitor in an RC circuit to 90% of full capacity given the following values of resistance and capacitance: R = 100 ohms, C = 1000 pF.

This is like a first order kinetics problem: the charge dissipation rate is proportional to the stored charge. We should expect exponential charge (or discharge). The applicable eqn. is:

\[ v_{\text{cap}} = V_i \times (1 - e^{-t/\tau}) \]

which can be “rearranged” (if you would like):

\[ e^{-t/\tau} = -\frac{(v_{\text{cap}} - V_i)}{V_i} \]

\[ \tau = \frac{t}{\ln((E_i - e_{\text{cap}})/E_i)} \]

btw, the RC term is known as the time constant, \( \tau \), for the RC circuit:

\[ \tau = RC = (100 \ \Omega)(1,000 \times 10^{-12} \ F) \]

\[ \tau = 1.00 \times 10^{-7} \ \text{sec} = 0.100 \ \mu\text{sec} \]

We want to know when the voltage in the capacitor rises to 90% (or 0.90) of the applied voltage, \( E_i \). So, plug in 0.90 for \( e_{\text{cap}} \) and 1 for \( E \) and substitute in the known value for \( \tau \) (i.e. RC):

\[ t = 2.3 \times 10^{-7} \ \text{sec or 0.23} \ \mu\text{sec} \]

Note: it will always take 2.303*\( \tau \) to charge a cap to 90% of the applied voltage. It will also take 2.303*\( \tau \) to discharge a cap to 10% of the initial voltage. In one \( \tau \) unit of time we achieve 37% charging (or discharging). You should be comfortable with this standard ln or exp math.

A BIT HARDER
2. A 100 mF capacitor is connected in series with a 10.0 \( \Omega \) resistor. This combination is connected in parallel with a 25.0 \( \Omega \) resistor. Both branches are then connected in parallel to a 4.50 V battery that can be switched on and off. The capacitor starts off fully discharged.

(a) What is the time constant in both branches when the switch is closed?
(b) What is the maximum charge that the capacitor can attain after the switch is closed?
(c) When will the voltage drop across the 10.0 \( \Omega \) resistor be equal to 1.50 V after the switch is closed?
(d) If the switch is opened, what will be the value of the new time constant and in which direction will the current flow through the 10.0 \( \Omega \) resistor?
(e) If the switch is opened after the capacitor is fully charged, how long will it take for there to be only one electron on the capacitor?
(f) If the switch is opened after the capacitor is fully charged, how long will it take for the voltage across the 25.0 \( \Omega \) resistor to drop to 1.50 V?

(answer on next page)
I “borrowed this question from PhyscsNet. Answers and solutions to this question are here: http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/EM_Problems/RC-Circuit-Problem/RC-CircuitMain.html

3. A typical RC circuit can be used as either a low pass or a high pass filter.

At the right you will find a table of capacitive reactance, frequency, impedance, and columns for “rejection ratios”. Data is for a RC circuit with $R = 10 \, \text{k}\Omega$ and a capacitance of $0.010 \, \mu\text{F}$. Remember $R/Z$ and $X_c/Z$ represent the ratio of voltage output over voltage input for the high pass side and low pass side of this filter respectively. Complete the calculations for the last two columns.

<table>
<thead>
<tr>
<th>$X_c$</th>
<th>Freq. (Hz)</th>
<th>$Z$</th>
<th>$R/Z$</th>
<th>$X_c/Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.59 x$10^{-1}$</td>
<td>1.00$x10^{18}$</td>
<td>10,000</td>
<td>1.000000</td>
<td>0.000</td>
</tr>
<tr>
<td>1.59 x$10^{-11}$</td>
<td>1.00 x$10^{16}$</td>
<td>10,000</td>
<td>0.999998</td>
<td>1.592E-03</td>
</tr>
<tr>
<td>1.59 x$10^{-13}$</td>
<td>1.00 x$10^{14}$</td>
<td>10,126</td>
<td>0.987570</td>
<td>0.1572</td>
</tr>
<tr>
<td>1.59 x$10^{-15}$</td>
<td>1.00 x$10^{12}$</td>
<td>159,469</td>
<td>0.062708</td>
<td>0.9980</td>
</tr>
<tr>
<td>2.65 x$10^{-6}$</td>
<td>6.00 x$10^{1}$</td>
<td>265,447</td>
<td>0.037672</td>
<td>0.9993</td>
</tr>
<tr>
<td>1.59 x$10^{-7}$</td>
<td>1.00 x$10^{0}$</td>
<td>15,915,497</td>
<td>0.000628</td>
<td>1.000</td>
</tr>
<tr>
<td>1.59 x$10^{-9}$</td>
<td>0.01</td>
<td>1591549431</td>
<td>0.000006</td>
<td>1.000</td>
</tr>
</tbody>
</table>

a. Across which circuit element (the resistor or the capacitor) would you monitor the signal if the circuit was being used as a HIGH PASS filter? Across the resistor b/c $V_{out} \sim V_{in}$ at high freq

b. A typical proton Nuclear Magnetic Resonance spectrometer (with a magnetic field strength of 4.69 Tesla) uses radio frequency (rf) radiation to probe the structure of molecules. The wavelength of the rf is 1.50 meters, what frequency is this?

\[ \nu = \frac{c}{\lambda} = 2.00 \times 10^{18} \text{ Hz or 200 MHz} \]

c. What energy does this radiation have?

\[ E = h\nu = 1.33 \times 10^{-25} \text{ J} \]

d. Based upon the frequency found above, could the RC circuit described above be used to filter out a 60 Hz line noise from the NMR signal? Look at the numbers above and justify your response. $R/Z = 1.00000$ for 100 MHz signal and 0.037672 for 60 Hz (see the R/Z column). Since 200 MHz is even higher frequency, essentially all of the signal would pass. What fraction of the 60 Hz noise would pass? 0.037672 for 60 Hz (so nearly 4% would pass)

e. How could the RC circuit be altered to improve the elimination of 60 Hz noise from the rf signal? Useful equations are: Reduce the value of R, reduce the value of C, or reduce the value of both. Try reducing R, C, and both by a factor of 10 to test the results.

\[ X_c = \frac{1}{2\pi fC} \quad Z = (R^2 + X_C^2)^{1/2} \]